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# VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD 

B.E. II Year (E.E.E.) I-Semester Supplementary Examinations, May/June-2017

## Partial Differential Equations and Numerical Methods

Time: 3 hours
Max. Marks: 70
Note: Answer ALL questions in Part-A and any FIVE from Part-B
Part-A (10 X $2=20$ Marks)

1. Find the value of $b_{1}$ for the Fourier series $f(x)=\frac{1}{4}(\pi-x)^{2}, 0<x<2 \pi$
2. Define Dirichlet's Conditions.
3. Form PDE by the elimination of arbitrary constants for the following equation $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$
4. Solve $(y-z) p+(x-y) q=z-x$
5. Solve the equation $\frac{\partial^{2} z}{\partial x^{2}}-2 \frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=0$ by the method of separation of variables.
6. Show that $\mathrm{u}=\mathrm{x}^{3}-3 x \mathrm{y}^{2}+3 \mathrm{x}^{2}-3 \mathrm{y}^{2}+1$ is a solution of Laplace equation.
7. State Lagrange's formula for unequal intervals.
8. Write Regula-Falsi iteration formula to find a root of the equation.
9. Prove that $\mathrm{z}\{\cos \mathrm{n} \theta\}=\frac{z(z-\cos \theta)}{z^{2}-2 z \cos \theta+1} i f|z|>1$
10. Write the linear property of Z -transforms.

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\begin{aligned}
& \text { Part-B }(5 \times 10=50 \text { Marks) } \\
& \text { (All bits carry equal marks) }
\end{aligned}
$$

11. a) Find the Fourier series to represent the function $f(x)$ given by

$$
f(x)=\left\{\begin{array}{cc}
x, & 0 \leq x \leq \pi \\
2 \pi-x, & \pi \leq x \leq 2 \pi
\end{array}\right.
$$

b) Find the fourier series for the function $f(x)=x-x^{2},-1<x<1$
12. a) Solve $z^{2}\left(p^{2}+q^{2}+1\right)=a^{2}$
b) Solve $2 z x-p x^{2}-2 q x y+p q=0$ by Charpit's method.
13. a) A tightly stretched string with fixed end points $x=0$ and $x=\ell$ initially in a position given by $y=y_{0} \sin ^{3}(\pi x / \ell)$ If it is released from rest from this position. Find the displacement $y(x, t)$.
b) Solve the equation $\frac{\partial u}{\partial t}=\frac{\partial^{2} u}{\partial x^{2}}$ with boundary conditions $u(\mathrm{x}, 0)=3 \sin (\mathrm{n} \pi x / \ell)$, $\mathrm{u}(0, \mathrm{t})=0, \mathrm{u}(\ell, \mathrm{t})=0$ where $0<\mathrm{x}<\ell, \mathrm{t}>0$
14. a) Find the first, second and third derivatives of the function tabulated below at the point $\mathrm{x}=1.5$

| $\mathrm{X}:$ | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{~F}(\mathrm{x})$ | 3.375 | 7.0 | 13.625 | 24.0 | 38.875 | 59.0 |

b) Use Runge-Kutta method of $4^{\text {th }}$ order to find $y$ when $x=1.2$ in steps of 0.1 given that $\frac{d y}{d x}=x^{2}+y^{2}$ and $y(1)=1.5$
15. a) Find the inverse $Z$ - Transform of $\frac{z^{2}+2 z}{(z-1)(z-2)(z-3)}$
b) Solve the difference equation $y_{n+2}+6 y_{n+1}+9 y_{n}=2^{n}$ given $y_{0}=y_{1}=0$
16. a) Expand $\pi x-x^{2}$ in a half range sine series in the interval $(0, \pi)$
b) Solve $p^{2}-q^{2}=x-y$
17. Answer any two of the following:
a) Solve the Heat equation $\left(\frac{\partial u}{\partial t}=c^{2} \frac{\partial^{2} u}{\partial x^{2}}\right)$ using method of separation of variables.
b) Describe the procedure for obtaining first and second derivative of a function by Numerical methods.
c) State and prove convolution theorem for Z-transforms.

